Statement:

PSU MATH 140 EXAM1

1. Suppose that lim (x -> 1+) f(x) = 4 = lim (x -> 1-) f(x), and f(1) = 7. Which one of the following statements is true?

   a) f is continuous but not differentiable at x = 1.
   b) f has a vertical asymptote at x = 1.
   c) f has a jump discontinuity at x = 1.
   d) f has a removable discontinuity at x = 1.
   e) f is differentiable at x = 1.

The graph of a function f(x) is pictured below. The tangent line to the graph at x = -2 is drawn, and the equation of the tangent line is given in the picture, but no formula for f(x) is given. Which one of the following statements must be true?

   a) f(-2) = 4, f(0) = 2
   b) f'(-2) = -2, f(-2) = 2
   c) f'(-2) = -1, f(-2) = 4
   d) f'(-2) = 1, f'(2) = 1
   e) f'(-2) = -1, f'(2) = -1

[The graph includes a curve with a tangent line represented by the equation y = -x + 2, and a marked point at x = -2.]

Result:

Question 1: Analysis of the Limits and Continuity of $f(x)$

We are given:
$\lim_{x \to 1^+} f(x) = 4$,
$\lim_{x \to 1^-} f(x) = 4$,
$f(1) = 7$.

Step 1: Continuity at $x = 1$
For a function to be continuous at $x = 1$, the following must hold:
$\lim_{x \to 1} f(x)$ exists.
$f(1)$ is defined.
$\lim_{x \to 1} f(x) = f(1)$.

Here:
$\lim{x \to 1^+} f(x) = \lim{x \to 1^-} f(x) = 4$, so $\lim_{x \to 1} f(x) = 4$.
$f(1) = 7$, which is not equal to $\lim_{x \to 1} f(x)$.

Thus, $f(x)$ is not continuous at $x = 1$.

Step 2: Type of Discontinuity
Since the left-hand and right-hand limits are equal (4) but differ from $f(1) = 7$, this is a removable discontinuity. The function could be made continuous by redefining $f(1) = 4$.

Step 3: Differentiability
A function must be continuous to be differentiable. Since $f(x)$ is not continuous at $x = 1$, it cannot be differentiable there.

Correct Answer:
The correct statement is:
d) f has a removable discontinuity at $x = 1$.

Question 2: Tangent Line and Derivatives

We are given:
The tangent line at $x = -2$ is $y = -x + 2$.
The slope of the tangent line is the derivative of $f(x)$ at $x = -2$, i.e., $f'(-2) = -1$.
The point of tangency is $(-2, f(-2))$, and substituting into the tangent line equation gives:

$$
f(-2) = -(-2) + 2 = 4.
$$

Step 1: Verify $f'(-2)$ and $f(-2)$
From the tangent line:
The slope is -1, so $f'(-2) = -1$.
The value of the function at $x = -2$ is $f(-2) = 4$.

Step 2: Eliminate Incorrect Options
Option a: $f(-2) = 4$ is correct, but $f(0) = 2$ is not given or verifiable.
Option b: $f'(-2) = -2$ is incorrect (the slope is -1).
Option c: $f'(-2) = -1$ and $f(-2) = 4$ are both correct.
Option d: $f'(-2) = 1$ is incorrect (the slope is -1).
Option e: $f'(-2) = -1$ is correct, but $f'(2) = -1$ is not verifiable from the graph.

Correct Answer:
The correct statement is:
c) $f'(-2) = -1, f(-2) = 4$